Problem Solving

The aim of this document is to define the Oxley College approach to explicitly teach problem solving within mathematics. For more detail on a lot of the ideas see [*Polya: How to Solve It*](https://notendur.hi.is/hei2/teaching/Polya_HowToSolveIt.pdf)

The Problem Solving Cycle

[*How to Solve It*](https://en.wikipedia.org/wiki/How_to_Solve_It) suggests the following steps when solving a mathematical problem:

**Understand the problem**

**Make a plan**

**Carry out the plan**

**Look back**

Strategies

What follows is a list of strategies that might help in solving problems, sorted loosely by where they appear in the problem solving cycle. They may appear twice, such as the **Draw a picture** or **Look at the end** strategy, because it helps in different ways at different stages of the problem solving cycle.

The descriptions for each category are posed as questions that can be asked of the students or students can ask themselves.

*Understand the problem*

**Make sense of the words**. Do you understand all the words in the problem? Can you restate the problem in your own words? Can you describe the problem to a friend without re-reading it?

**Draw a picture.** Can you think of a picture or a diagram that might help you understand the problem? Try drawing the diagram as you read through the problem, one sentence at a time.

**Model the problem.** Can you simulate the problem scenario somehow? Can you act out the problem scenario (with people, objects or diagrams)?

**Look at the end**. What, exactly, are you being asked to find? What will the answer look like? Is it a number, an equation, a shape? *Respice finem* (“look/consider at the end”) is the Latin for this technique – it has been around for a while!

**List the givens**. What information is *given*? Can you list them all? Can you separate them? They may be *constraints* (things that have to stay true, often where a variable has a particular value or a minimum or maximum value) or they might be *conditions* (situations / times where the variables have particular values) or *relationships* between the variables (often described as an equation).

**Define the Variables**. Can you define the variables? What units are they measured in? Can you introduce simple, clear notation? Are there notation conventions you can use (such as A for a point, AB for a line)? Notation is important – you don’t start a problem by writing the numbers down in Roman numerals, or as words, for example. You don’t name speed *b* and time *w* unless you have some compelling reason.

**Ask a question / find out more.** Do you need to ask a question to help understand the problem? Do you have enough information to find a solution? Do you need to do some research to begin the problem?

*Make a plan*

**Find a similar problem**. Have you seen a similar problem before?

**Find a simpler problem**. Can you solve a simpler problem? A more specific problem (such as substituting easy values instead of solving for variables)? If you remove some parts of the problem, or make the numbers nicer, is it easier to solve?

**Find a subproblem.** Can you break the problem into smaller subproblems? Can you solve some of these instead?

**Find an analogous problem**. Can you restate the problem into an equivalent problem that is easier to attack? For instance, the problem “Find where the lines and intersect” is analogous to the problem “Solve

*When finding a different problem, the aim is always to find an easier problem.*

**Form an equation**. Are there words in the problem that might mean some quantities are equal? (look for words like: *is, are, was, equals, will be, gives, results in, yields, sold for, cost*)Can you translate your variables into equations? How are your variables connected? *Setting up an equation is like translating from normal language into the language of formulas.*

**Use a formula**. Do you know any formula that might be useful?

**Guess and check.** Can you make a guess? Can you check to see if it is right? Can you make a series of guesses and see which are closer to your answer?

**Organise**. Can you make an orderly list of the data? Can you put the data into a table, or similar, to help look for patterns? Is there a natural way to organise the data?

**Look for patterns.** Can you see any patterns?

**Special cases**. Can you check very simple cases? Can you check extreme cases?

**Eliminate possibilities**. What is definitely *not* the answer? Can you eliminate everything *above* or *below* a certain point?

**Variable reduction**. Do you need all the variables you have defined, or do some depend on the values of others? Can you use the givens to help express some variables using other variables?

**Use everything**. Did you use all the data / givens? Is there something that should influence your answer that you haven’t used?

**Use symmetry**. Can you reduce the problem by using symmetry? Are parts of the problem similar to other parts of the problem?

**Draw a picture**. Can you represent the problem with a diagram or picture? Can you represent only the important part of the problem with a picture?

**Model the problem**. Can you build a model of the problem (physical, computational, diagram)? Can you simulate the problem scenario somehow? Can you act out the problem scenario?

**Work backwards**. If you had the answer, what would have been the last step? What do you need to know to get the solution? If you had the answer, what else would you know?

**Look at the end**. What, exactly, are you trying to find? What kind of thing is it (number / shape / etc)? What units should it have?

*Carry out the plan*

**Check each step**. Can you see clearly that the step is correct? Can you prove that it is correct? Can someone else check?

*Look Back*

**Check the answer**. Can you check the solution? Can you check your argument?

**Other ways**. Is there another way to solve the same problem? Can you find an easier path to the solution?

**Other applications**. Can this method be used to solve other problems? Can the method be generalised?

References:

A nice summary of Polya’s framework: <http://web.mnstate.edu/peil/M110/Worksheet/PolyaProblemSolve.pdf>

The original work, [*Polya: How to Solve It*](https://notendur.hi.is/hei2/teaching/Polya_HowToSolveIt.pdf)

On translating words in maths problems: <http://www.purplemath.com/modules/translat.htm>